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Theoretical Bound of CRLB for Energy Efficient Technique of RSS-Based Factor Graph Geolocation

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Abstract. To support the increase of wireless geolocation development as the key of the technology in the future, this paper proposes theoretical bound derivation, i.e., Cramer Rao lower bound (CRLB) for energy efficient of received signal strength (RSS)-based factor graph wireless geolocation technique. The theoretical bound derivation is crucially important to evaluate whether the energy efficient technique of RSS-based factor graph wireless geolocation is effective as well as to open the opportunity to further innovation of the technique. The CRLB is derived in this paper by using the Fisher information matrix (FIM) of the main formula of the RSS-based factor graph geolocation technique, which is lied on the Jacobian matrix. The simulation result shows that the derived CRLB has the highest accuracy as a bound shown by its lowest root mean squared error (RMSE) curve compared to the RMSE curve of the RSS-based factor graph geolocation technique. Hence, the derived CRLB becomes the lower bound for the efficient technique of RSS-based factor graph wireless geolocation.

1. Introduction

Wireless geolocation technologies have played crucial role in the human life. This is because these technologies are significantly required to enable a lot of location-aware services and applications. Hence, the research in wireless geolocation filed is continuously increasing, mainly since two decades ago, when federal communication committee (FCC) issued the requirement for all wireless equipment to support enhanced 911 (E-911)[1]. However, the wireless geolocation technologieshave been utilized since long time ago before the issuing of the above FCC requirement, i.e., in military application such as the radar to detect the approaching objects, the public service of flights services in managing the aeroplane traffic on the air, as well as the introducing of global position system (GPS)[2]. Currently, the utilization of wireless geolocation technologies are incorporated to vast area of services and applications, i.e., wireless charging, vehicle navigation, precision agriculture, elderly and children monitoring, finding victims of disaster, andillegal radio monitoring[3] -[4].

A lot of techniques for wireless geolocation technologies have also been invented with various approach, i.e., least square[5], methods of moment (MOM)[6], Gauss Newton (GN)[7],[8], and factor graph techniques[9], [10]. We notice that the factor graph based of geolocation is one of the most promising resulting high accuracy of detection and low computation of complexity. This is because the message passing algorithm utilized in factor graph framework efficiently and effectively coordinates the stochastic information obtained from the measurement to estimate the position location of wireless devices. The development of factor graph wireless geolocation technique itself begins since two years after the factor graph framework was firstly introduced in [11]. That is when [12] first utilizes factor graph framework to process the stochastic information of time of arrival (TOA) obtained from the measurement to estimate the position of wireless device. After that time, the development of factor graph for wireless geolocation techniques is significantly increasing where all of those techniques exploit the stochastic information of extracted signal metric of the waveforms, e.g., TOA, time differential of arrival (TDOA), direction of arrival (DOA), received signal strength (RSS), and differential signal strength (DRSS). The basic configuration of wireless geolocation technique with factor graph algorithm is shown in Fig. 1. It should be noted that the monitoring spots are only required by RSS-based technique.

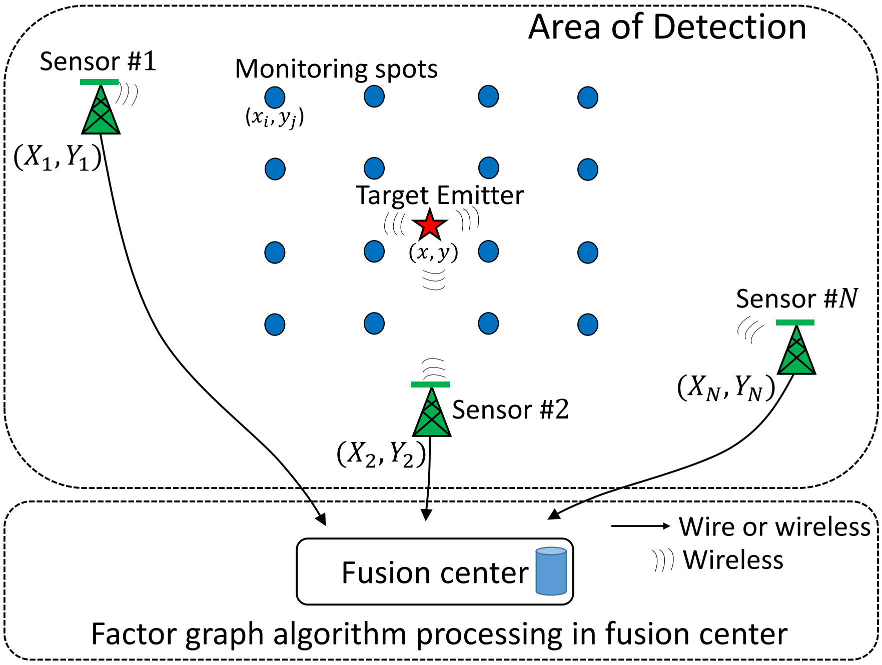

Figure 1. The basic configuration of wireless geolocation with factor graph algorithm.

To ensure the effectiveness and to increase the possibility of the innovation and development of the factor graph based wireless geolocation techniques, the theoretical bound is necessary to be derived. It is well known that the most widely used and easiest bound for any estimation technique is Cramer Rao lower bound (CRLB). Hence, the CRLB for factor graph for TOA[7], TDOA [7], and DOA-based factor graph wireless geolocationtechniques [13] are derived. Furthermore, the introduction of K sample into the CRLB for thosefactor graph techniques is derived in our previous work [14], [15].However, the CRLB for RSS [10] and DRSS-based [16] factor graph wireless geolocation have not been derived yet. Hence, this paper derives the CRLB for RSS-based technique, while the CRLB for DRSS-based technique is left for future work.

2. Model System

Let $\mathbf{x} = [x \;\; y]^T$ denotes the target position of wireless device in X-Y coordinate system, where$[x \;\; y]$ and $[\cdot]^T$ are the row vector of the position coordinate and transpose function, respectively. While $\mathbf{X} = [X_i Y_i]^T$ denotes the position of sensor or receiver, where $i, i = \{1, 2, \dots, N\}$,is the sensor index number, while$N$ denotes the total number of sensors. In the RSS-based wireless geolocation system problem, the target position $\mathbf{x}$is estimated with the processing of the RSS value obtained from the measurement. The RSS value itself is the power signal strength received by the sensor or radio receiver. It should be noted that the RSS measurement has been common available in any radio receiver such as smart phone, wireless access point, and other radio receiver devices.

To estimate the target position $\mathbf{x}$, the relationship between the RSS samples ofP_i in each sensor and the target position $\mathbf{x}$ should be available.It is assumed that the P_i follows Gaussian distribussion due to the accumulation effect of many independent factors as mentioned in [10],[9]. This relationship has been derived in [10] for RSS-based factor graph wireless geolocation utilizing the finger print approach[1] expressed as

$$a_{x_i} \cdot x + a_{y_i} \cdot y + a_{p_i} \cdot P_i = c, \tag{1}$$

where a_{x_i}, a_{y_i}, and a_{p_i} are the coefficients of x, y, and P_i, respectively.Furthermore, c denotes a constant with the value of 1 as applied in [10].

It should be noted that the calculation of coefficients of (1) is conducted before the factor graph algorithm is ready for estimating the position of radio emitter. Furthermore, the value of coefficients in (1) for each i-th sensor are obtained by exploiting the measured RSS ofand the coordinate positionof monitoring spots,$P_{i,j}$ and (x_j, y_j), respectively, where$j$, $j = \{1, 2, \dots, M\}$, is the monitoring spots index number,and$M$is the total number of monitoring spots. In addition, the position and transmit power of monitoring spots are known. Then, the RSS of the signal transmitted by monitoring spots,having the training or pilot signal sent to the sensors, is measured in sufficient long duration to eliminate the error of measured RSS value. Hence, the use of the monitoring spots are for mapping the finger print of RSS value with certain pattern of position by applying least square formula as shown in [10] expressed as

$$\mathbf{A} = (\mathbf{B}^T \cdot \mathbf{B})^{-1} \cdot \mathbf{B}^T \cdot \mathbf{C}, \tag{2}$$

where$\mathbf{A}$is a column vector of a_{x_i}, a_{y_i}, and a_{p_i}, $\mathbf{B}$ is a matrix of x_j, y_j, and $P_{i,j}$. Then, $\mathbf{C}$ is the unit columns vector. Therefore, the equations of (1) having calculated coefficients for each sensorare obtained.

It is shown in [10] and [17]that four monitoring spots with forming square position surrounding the target are the optimal configuration as depicted in Fig. 1.Our work in [17] shows that the estimate accuracies with both area widths of $100 \times 100\ \mathrm{m}^2$and $200 \times 200\ \mathrm{m}^2$of monitoring spots is sufficient. Furthermore, the algorithm to choose four appropriate monitoring spots located surrounding the target also can be found in our work[18]. The detailed explanation of the RSS-based factor graph technique for wireless geolocation is not discussed in this paper to save the space, however, it can be found in [14] and [10]. Therefore, the next discussion directly goes through to the CRLB derivation for RSS-based factor graph wireless geolocation technique.

It is shown in [2], [7], [8], that the general equations of CRLB is expressed as

$$\mathrm{CRLB} = \sqrt{\mathrm{trace}(\mathbf{F}^{-1}(\mathbf{x}))}, \tag{3}$$

where$\mathbf{F}$ denotes the Fisher information matrix (FIM) as

$$\mathbf{F}(\mathbf{x}) = E\left[\left(\frac{\partial}{\partial \mathbf{x}} \ln p(P_i)\right)^2\right], \tag{4}$$

where$E[\cdot]$, $\frac{\partial}{\partial \mathbf{x}}$,and $p(\cdot)$denote the expectation function, the first derivation respect to $\mathbf{x}$, and the probability density function, respectively.Then, the close form of the CRLB formula with $\mathbf{J}$ being the Jacobian matrix,$\Sigma_P = \sigma_P \mathbf{I}_\mathrm{N}$ being the covariance matrix of RSS sample, $\mathbf{I}$being the identity matrix,

[1]It should be noted that the RSS-based wireless geolocation problem, in general, contains two approach, i.e., (1) finger print and (2) model approach.

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Kbeing the measured sample, and $(\cdot)^{-1}$being inverse matrix, is derived from (3) and (4) in our work [14],[15], as

$$\mathrm{CRLB} = \sqrt{\mathrm{trace}\left((\mathbf{J}^{\mathrm{T}}\Sigma_P^{-1}\mathbf{J})K\right)^{-1}}. \tag{5}$$

Then, to obtain the CRLB for RSS-based factor graph geolocation technique, $\mathbf{J}$ should be derived according to (1), where $\mathbf{J} = \frac{\partial P_i}{\partial \mathbf{x}}andP_i = \frac{c}{a_{p_i}} - \frac{a_{x_i}}{a_{p_i}}x - \frac{a_{y_i}}{a_{p_i}}y$.Finally, the Jacobian matrix for RSS-based factor graph wireless geolocation is obtained as

$$\mathbf{J} = \left[-\frac{a_{x_i}}{a_{p_i}} \quad -\frac{a_{y_i}}{a_{p_i}}\right]. \tag{6}$$

3. Numerical Results

The computer simulation is conducted to validate the derivation of the CRLB utilized as the lower bound of the RSS-based factor graph wireless geolocation technique. Then, 10,000 times of trials are repeated for smoothing to the curves of the accuracy of the CRLB and the RSS-based factor graph technique.Furthermore, the accuracy itself is defined with the performance of root mean square error (RMSE).Note that three sensors are applied with located at $(100, 0), (1100, 0),$ and $(600, -1000)$m. Then, the target locations are randomly chosen in the centre of area width of $600 \times 600\ \mathrm{m}^2$. In addition, two scenarios of monitoring spots area width are $100 \times\ 100\ \mathrm{m}^2$and $200 \times 200\ \mathrm{m}^2$. Finally, the signal to noise ratio (SNR) values for testing the algorithm and formula are 0 to 45 dB with the step of 5 dB.

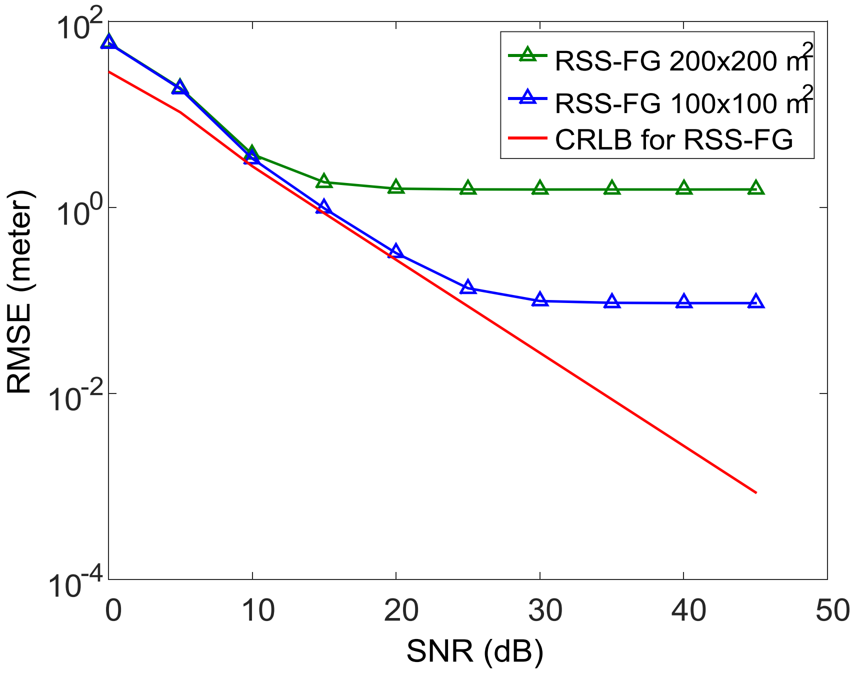

Figure 2. The RMSE of RSS-based factor graph and its CRLB. Note that RSS-FG in the figure stands for RSS-based factor graph technique.

As mentioned above, the detailed of RSS-based factor graph wireless geolocation algorithm can be found in [14], [10], hence we do not present in this paper. Furthermore, 100 iterations are conducted in each trial of the factor graph algorithm simulation. It is assumed that the RSS signal is suffered from path-loss, where in the simulation uses exponential path-loss model with $d_0 = 100$ m being distance reference, $n_p = 3$ being the exponential value, and $f_c = 1$ GHz being the carrier frequency.

Fig. 2 shows that in term of the accuracy of RMSE, the derived CRLB has been the lower bound for both of the RSS-based factor graph wireless geolocation techniques with 100×100 m^2 and 200×200 m^2 of monitoring spots width area. It is also shown that the larger area of monitoring spots the lower of the accuracy is obtained. Furthermore, the accuracy of the factor graph algorithm becomes stagnant above 15 dB and 30 dB for monitoring spots width area of 200×200 m^2 and 100×100 m^2, respectively, while the higher SNR the higher accuracy of the CRLB with linear curve shape.

It is also shown in Fig. 3 that the CRLB curve accuracy located at the lowest RMSE of each iteration which means the CRLB is the most accurate served as the lower bound. It should be noted that the lower of RMSE, the higher accuracy of the estimate. Furthermore, Fig. 3 depicts that the iteration of the algorithm of factor graph converges about at 16-th iteration where the RSS-based factor graph with lower area width of monitoring spots outperforms the wider one.In addition, it is shown in [10] that the computational complexity of the RSS-based factor graph wireless geolocation technique is linearly proportional to N. Hence, the CRLB becomes the lower bound of the energy efficient technique of RSS-based factor graph wireless geolocation.

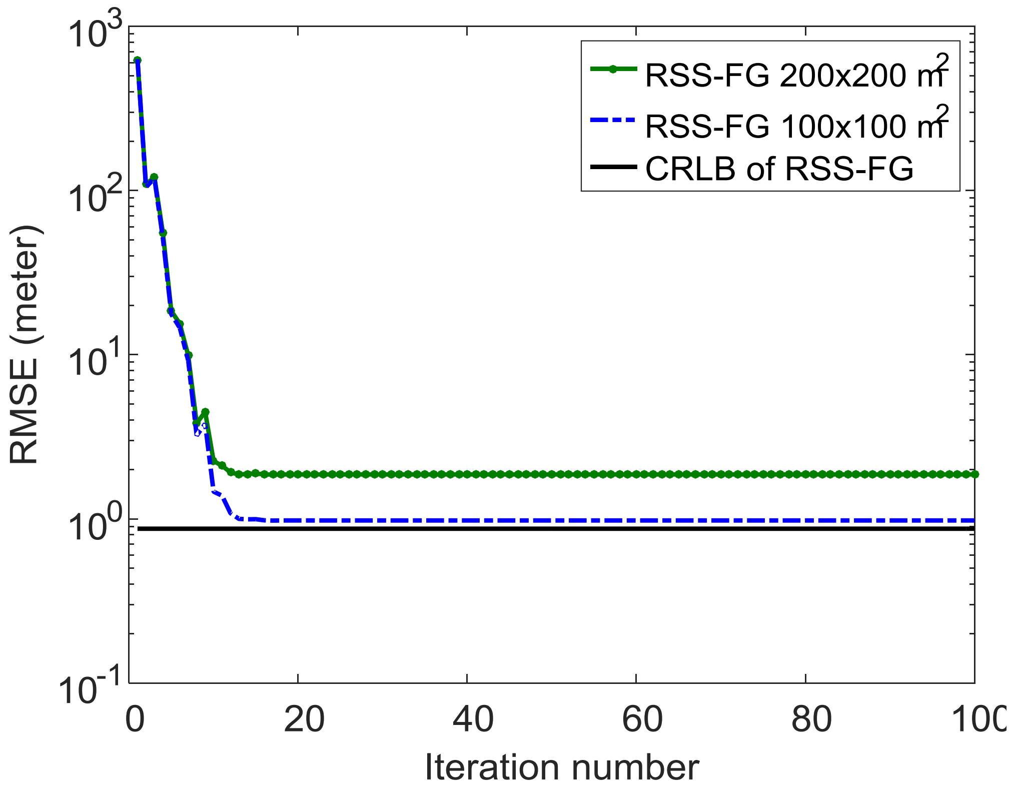

Figure 3. The RMSE of RSS FG and CRLB for RSS FG. Note that FG stands for factor graph.

4. Conclusion

The derivation of CRLB for RSS-based factor graph wireless geolocation has been presented in this paper. It should be noted that the key of the derivation is lied on Jacobian matrix inside the CRLB formula. Hence, the Jacobian matrix is derived from the expression of the relationship between RSS with the coordinate position utilized by the low complexity of RSS-based factor graph wireless geolocation algorithm. Finally, the simulation results shows that the derived CRLB has the highest accuracy compared to the RSS-based factor graph wireless geolocation technique. It means that the derived CRLB is the theoretical lower bound of the energy efficient of RSS-based factor graph wireless geolocation technique.

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